**394.1 Advanced Corporate Finance - Valuation**

**Homework Assignment 3 – due on Monday October 21.**

**Please submit one Word or PDF document per group, and indicate group members’ names on the first page.**

**The assignment has a total of 150 points.**

**Question 1**:

We have the following information on three risk-free bonds:

Current Value Year 1 payoff Year 2 payoff Year 3 payoff

Bond A 96 100 0 0

Bond B 94.2360 10 10 90

Bond C 104.5101 20 20 80

**1a)** **(5 points)** Consider a new bond, Bond D, which has the same payoff as Bond B in years 2 and 3, but has a payoff of zero in year 1. Describe the tracking portfolio of Bond D, and find its current market value (Hint: You do not need to set up three equations in three unknowns to solve this problem).

**1b)** **(5 points)** Similar to part (a), consider a new bond, Bond E, which has the same payoff as Bond C in years 2 and 3, but has a payoff of zero in year 1. Describe the tracking portfolio of Bond E, and find its current market value (Same hint as above applies).

**1c)** **(5 points)** Consider a zero-coupon bond that pays 100 in year 2 and zero in years 1 and 3. Describe the tracking portfolio of this zero-coupon bond in terms of Bonds D and E in parts (a) and (b). Find the current market value of the zero-coupon bond, and its annual yield.

**1d) (5 points)** Consider a zero-coupon bond that pays 100 in year 3 and zero in years 1 and 2. Describe the tracking portfolio of this zero-coupon bond in terms of Bonds D and E in parts (a) and (b). Find the current market value of the zero-coupon bond, and its annual yield.

**1e) (5 points)** You have a project that costs 400 to invest in today. The project pays 150 in year 1, 200 in year 2, and 250 in year 3. Calculate the NPV of the project (you can use any method or any of the findings from earlier parts of the question). If the NPV is positive, describe how you can form a hedged position by investing in the project and simultaneously in bonds so that you obtain the NPV of the project as an upfront arbitrage at the time you invest (here you can use any of bonds A, B, C, D, E, and the two zero-coupon bonds that you priced in parts (c) and (d)).

**Question 2:**

Exxon is evaluating an oil well that will produce oil for two years. In year 1, the company plans to produce 100,000 barrels of oil, and the production cost will be $20 per barrel. In year 2, it plans to produce 50,000 barrels of oil, but the production cost will go up to $30 per barrel. The well will be fully depleted in two years, and it has no salvage value afterwards. The forward price of oil to be delivered one year from now is $80, and the forward price oil to be delivered two years from now is $90. The risk-free rate for all maturities is 5%.

**(a) (5 points):** Find the certainty equivalent cash flows of the oil well in years 1 and 2. Find the current market value of the well using the certainty equivalents.

**(b) (5 points):** Describe the tracking portfolio of the oil well, i.e., a portfolio of financial assets that synthetically replicates the cash flows from the well.

**(c) (5 points):** Suppose that the production plans for year 1 are as above, but there is some uncertainty about production in year 2. Specifically, a technical report indicates that, after the first year production of 100,000 barrels, the well will have either 25,000 or 75,000 barrels of oil left. The two possibilities have equal likelihood. Given this information, indicate whether you need to change your analysis of the valuation of the well in part (a), and if so, how.

**Question 3 (15 points):** You hold an American call option with a $30 strike price on a stock that currently sells at $35. The risk-free rate of interest is rf. Compare the cash flows at expiration from (i) exercising the option now and putting the $5 proceeds in a bank account that pays the risk-free rate until the expiration date and (ii) holding onto the option until expiration, selling short the stock today, and placing $35 you receive into the same bank account.

**Question 4 (20 points):** The present price of an equity share of Strategy Inc. is $50. The stock follows a binomial process where each period the stock either goes up 10 percent or down 10 percent. Compute the market value of an American put option on Strategy Inc. stock with a strike price of $50 and two periods to expiration. Assume Strategy Inc. pays no dividends over the next two periods. The risk-free rate is 2 percent per period.

**Question 5 (25 points):** The XYZ firm can invest in a new DRAM chip factory for $425 million. The factory, which must be invested in today, has cash flows two years from now that depend on the state of the economy. The cash flows when the factory is running at full capacity are described by the following tree diagram:

Year 0 Year 1 Year 2

Very good

$1 billion

Good

Medium

$200 million

Bad

Very bad

-$500 million

In year 1, the firm has the option of running the plant at less than full capacity. In this case, workers are laid off, production of memory chips is scaled down, and the subsequent cash flows are half of what they would be when the plant is running at full capacity.

Investing $1 in the market portfolio in year 0 results in the following payoffs:

Year 0 Year 1 Year 2

$1.22

$1.10

$1.04

$1.00

$0.95

$0.91

Assume that the risk-free interest rate is 5 percent per year. Compute the project’s present value **(a)** without the option to scale down, and **(b)** with the option to scale down.

**Question 6:**

A vacant lot has been zoned to construct today either 50 offices or 50 apartments.  Assume that:

1) Construction must occur immediately and cannot be delayed.

2) Construction takes no time. Therefore, the offices or the apartments we build today can be sold or rented out immediately.

3) Construction costs are $75 per office or $90 per apartment.

4) The current price of offices is $100 and the current price of the apartments is $105.

5) The rental rate is $10 per office and also $10 per apartment (net of expenses, to be received at the end of the year).

6) Next year,

a) If the real estate market booms, offices will sell for $140 and apartments for $125.

b) If the real estate market does poorly, offices will sell for $85 and apartments for $100.

7) The risk-free interest rate is 12%.

**6a) (5 points)** What is the value of the vacant lot?

**6b) (15 points)** Suppose now that, in addition to the previous information, a new construction technology becomes available. This technology allows building offices today that can be converted into apartments next year. The cost of converting each office unit into an apartment unit is $10 (to be paid next year, and only if an office is actually converted into an apartment). The opposite conversion is not possible (i.e., if we build apartments today, they cannot be converted into offices next year). Also, the conversion applies only to the particular offices in consideration for building; it does not apply to existing offices traded in the market. Find the value of the lot now.

**6c) (5 points)** Suppose now that the cost of converting offices into apartments is $20 per unit (again, to be paid next year). What is the value of the lot now? Explain.

**Question 7:**

Firm X can choose between two technologies A and B to produce widgets.

* Technology A has low fixed costs but high variable costs. Adopting A will require the firm to pay today $20,000 and will allow firm X to produce in a year UP TO 100,000 widgets at a variable cost of $1.5 per widget. (Suppose that the production of widgets occur, instantaneously, at t=1.)
* Technology B has a high fixed cost but low variable costs. Adopting B will cost the firm today $60,000 and will allow the firm to produce in a year UP TO 100,000 widgets at a variable cost of $1 per widget.
* Suppose that the forward price for widgets is $1.8 per widget delivered next year and that the price of a widget next year can be either $1.2 or $2.2.
* Assume that *rf* =10% and that after producing the widgets at t=1, both technologies become totally obsolete (so that their salvage value is zero).

**7a) (10 points)** Suppose that both technologies commit the firm to produce 100,000 widgets. Which technology if any should Firm X choose? (Hint: you do not need to any real option analysis to answer this part).

7**b) (15 points)** Suppose now that the firm makes the technological choice at t=0, but can wait until t=1 to decide how many, if any, widgets to produce. Which technology should Firm X choose in this case? (Hint: you need to do a real option analysis to answer this part).